PHYSICAL REVIEW E, VOLUME 63, 025104(R)

Failure regimes in (1+1) dimensions in fibrous materials

I. L. Menezes-Sobrinho,[1] A. T. Bernardes,[2] and J. G. Moreira[3]
[1]*Departamento de Física, Universidade Federal de Viçosa, 36570-000 Viçosa, MG, Brazil*
[2]*Departamento de Física, Universidade Federal de Ouro Preto, 35400-000 Ouro Preto, MG, Brazil*
[3]*Departamento de Física, Instituto de Ciências Exatas, Universidade Federal de Minas Gerais, CP 702, 30123-970 Belo Horizonte, MG, Brazil*

(Received 31 August 2000; published 26 January 2001)

In this paper, we introduce a model for fracture in fibrous materials that takes into account the rupture height of the fibers, in contrast with previous models. Thus, we obtain the profile of the fracture and calculate its roughness, defined as the variance around the mean height. We investigate the relationship between the fracture roughness and the fracture toughness.

DOI: 10.1103/PhysRevE.63.025104 PACS number(s): 62.20.Mk, 64.60.Fr, 05.40.−a

I. INTRODUCTION

The fracture of a material under different loading conditions is a complicated phenomenon that continues to attract the attention of several physicists and engineers. The fracture follows a sequential process of nucleation, growth, and coalescence of numerous cracks. These cracks tend to create a rough profile which often can be described by self-affine scaling [1]. In the last decade, great theoretical and experimental efforts have been done trying to understand the process of formation and propagation of cracks in materials. A great number of studies have been devoted to this process from the point of view of statistical mechanics, which utilize concepts such as percolation, fractals, scaling law, etc. [1,2]. A variety of computational models for material fracture have been developed in recent years and provided interesting results [2–4]. However, the high degree of correlations between the constituents leads to a high computational cost. An alternative is to employ fiber bundle models introduced more than 40 years ago [5], in which bundles of unidirectional fibers form a system with low degree of correlations allowing the fracture process to be simulated in a large scale.

The fracture of the material can occur through a big crack which percolates the sample. In this case we say that the fracture is catastrophic. When the fusion of small cracks cause the fracture of the material, it is called shredding [6]. In the catastrophic regime the fracture profile is reasonably smooth, while in the shredding regime it is very rough. A parameter of easy physical interpretation used to characterize fracture surfaces is the roughness. It is defined as the variance around of the mean height of the fracture profile, and measures the complexity of the crack path. Thus, the rougher is the fracture profile the harder is the crack path. The roughness has a direct relation to the fractal dimension, which characterizes the fractal character of the surface fracture. Therefore, a high fractal dimension indicates a very rough fracture profile [1,7].

Another important parameter in the study of fracture is the fracture toughness. This quantity measures the amount of energy that a material can absorb before it fractures. The fracture toughness is intimately related to the amount of cracks that appear in the material [2]. Therefore, the larger the number of cracks in the sample more energy will be absorbed before catastrophic fracture occurs. There are several works in the literature which investigate the relationship between the fracture roughness and the fracture toughness [7–11].

In the present Rapid Communication, we simulated a model for fracture in fibrous materials which allows us to obtain the fracture profile of samples in contrast with previous models [6,12–16]. We calculate the profile roughness and used the obtained values to define the transition between two fracture regimes: catastrophic and shredding. We also investigate the relationship between the fracture roughness and toughness for fibrous materials.

II. MODEL

Our model consists of a (1+1) d bundle of N_0 parallel fibers all with the same elastic constant k. In order to simulate the height of the sample, the fibers are divided into η segments with the same length. The fiber bundle is fixed at both ends to two parallel plates. One plate is fixed, and in the other, a constant force F is applied, for example, by hanging a weight on it. This force is shared in the same amount, σ, on each fiber of the bundle, which undergo the same linear deformation $z=F/Nk$, where N is the number of unbroken fibers. When the deformation z reaches a critical value z_c, the failure probability of an isolated fiber is equal to 1. The failure probability of a fiber i is given by [6]

$$P_i(\delta,t)=\frac{\delta}{(n_i+1)}\exp\left[\frac{(\delta^2-1)}{t}\right], \tag{1}$$

where n_i is the number of unbroken neighboring fibers, $\delta=z/z_c=F/Nkz_c$ is the strain of the material, $t=K_BT/E_c$ is the normalized temperature, K_B is the Boltzmann constant, T is the absolute temperature, and E_c is the critical elastic energy. In this model, besides finding the failure probability of a fiber, we have to indicate in which segment it breaks. This segment is randomly selected and the probability of the fiber to break in it is given by

$$\phi_j(m_j)=\frac{(m_j+1)}{\zeta}, \tag{2}$$

where m_j is a vector which indicates how many times a segment j broke and $\zeta=\Sigma_j(m_j+1)$. The form of Eq. (2) simulates a concentration of tension near to the region where the fiber bundle is more weak.

At the beginning of the simulation, the bundle is submitted to a initial strain given by

$$\delta_0=\frac{z_o}{z_c}=\frac{F}{N_0 k z_c}. \tag{3}$$

At each time step of the simulation we randomly choose a fiber of a set of $N_q=qN_o$ unbroken fibers. The number q represents a percentage of fibers and allows us to work with any system size. Then, using Eq. (1) we evaluate the fiber failure probability P_i and compare it with a random number r in the interval [0,1). If $r<P_i$ the fiber breaks. We then choose a segment j in the fiber and evaluate its probability ϕ to break, using Eq. (2). If the probability ϕ is higher than the random number ρ the fiber breaks in the chosen segment. If not, we analyze the neighboring segments $(j+1)$ and $(j-1)$ and again, evaluate the probability ϕ. If the condition $\rho<\phi$ does not hold to neither of the neighboring segments, we return to the initial segment and test the condition $\rho<\phi$ for a new value of ρ. This process continues until the condition $\rho<\phi$ is true. Once defined the segment where the fiber breaks, we begin to test all neighboring unbroken fibers. The first segment tested in the neighboring unbroken fibers is the segment where the previous fiber broke. The failure probability P_i of these neighboring fibers increases due to the decreasing of n_i and a cascade of breaking fibers may begin. This procedure describes the propagation of a crack through the fiber bundle, which occurs in the perpendicular direction to the applied force. The process of propagation stops when the test of the probability does not allow rupture of any other fiber on the border of the crack or when the crack meets another already formed crack. The same cascade propagation is attempted by choosing another fiber of the set N_q. After all the N_q fibers have been tested, the strain δ is increased if some fibers have been broken. Since the force is fixed, the greater the number of broken fibers, the larger is the strain on the intact fibers and the higher is their failure probability. Then, another set of N_q unbroken fibers is chosen and the entire rupture process is restarted. The simulation terminates when all the fibers of the bundle are broken, i.e., when the bundle breaks apart. At this stage we consider the profile of the fracture and analyze its statistical properties.

III. RESULTS

Using the model described in the preceding section we simulated the fracture of a fibrous materials under a static force F. In these simulations we considered the elastic constant $k=1$, the critical deformation $z_c=1$, the number of segments $\eta=100$ and the normalized temperature $t=0.5$.

Figure 1 shows the fracture profile for a set of $N_0=600$ fibers and three different forces. Notice that, the lower the applied force the larger is the roughness of the fracture profile, i.e, more irregular is the crack path. For $F=4000$ only one crack propagates in the material and for $F=2800$ and $F=2000$ more than one crack crosses the material. It can be seen in Fig. 1, that the rupture of the sample begins in different segments. This occurs because we did not consider a deterministic starting notch in our simulations.

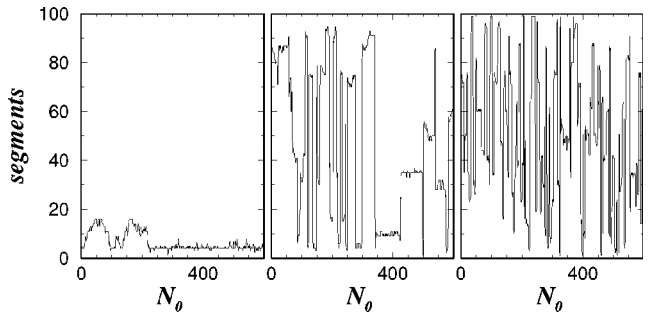

FIG. 1. Fracture profiles for three different forces. From left to right: $F=4000$, $F=2800$, and $F=2000$ arbitrary units. In this simulation we used a total of 600 fibers.

Figure 2(a) shows the results obtained for the roughness W as a function of the applied force F for different values of N_0. To measure the roughness of the fracture profile we used the method of the best linear least-square fitting described in Ref. [17]. In this plot we can observe that the roughness W decreases as the force F increases, and after a critical force F_c it stays constant. For each system size we have a characteristic value for F_c, which increases with the increase of the number of fibers N_0. This indicates that the greater the number of fibers in the bundle, the tougher is the sample. For $F>F_c$ the fracture is catastrophic, i.e., the breakage of a fiber induces the rupture of the whole bundle. In this case the bundle breaks with only one crack. In this region the crack propagates in the material with high speed, leading to a quite rapid rupture process. For $F<F_c$ the rupture of the bundle

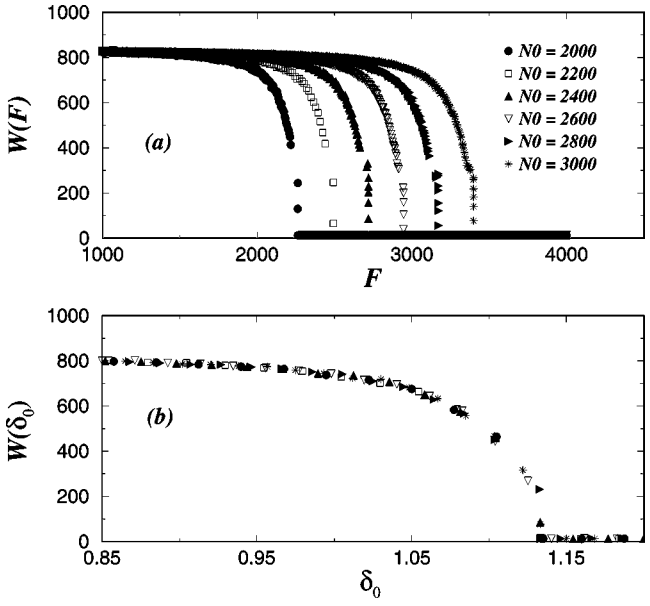

FIG. 2. (a) Roughness $W(F)$ as a function of the applied force F for different numbers of fibers N_0. The data were averaged over 1000 samples. (b) The x axis was converted to the initial strain $\delta_0=F/N_0$.

occurs due to the formation of small cracks, which weaken the bundle. Here, the crack speed tends to zero and a slow process of successive ruptures appears in the material.

In Fig. 2(b), the x axis was converted to the initial strain $\delta_0 = F/N_0$. The data are now found to collapse all on the same roughness-strain curve. From this curve we can find the critical initial strain δ_{0c}, which does not depend on the system size. Our results indicate $\delta_{0c} = 1.134$. We may assume that, for an initial strain greater than δ_{0c} just one crack provokes the rupture of the material and that, for a initial strain below δ_{0c} the rupture occurs through the fusion of small cracks.

The critical value δ_{0c} can be obtained analytically. The failure probability, Eq. (1), can be written as

$$P_i(\delta,t) = \frac{\Gamma(t,\delta)}{(n_i+1)}, \tag{4}$$

where $\Gamma(t,\delta)$ is defined as [6]

$$\Gamma(t,\delta) = \delta \exp\left[\frac{\delta^2-1}{t}\right]. \tag{5}$$

Using Eq. (4) we may observe that for $\Gamma(t,\delta) = 2.0$ the breakage of any fiber induces the rupture of the bundle with just one crack. Then, we can assume that there is a critical value for $\Gamma(t,\delta) = 2.0$ that defines the transition between two regimes. In the first regime, a catastrophic fracture occurs in the first attempt to break the bundle, while in the second one the rupture of the bundle occurs due to the formation of small cracks, which weaken the bundle. From Eq. (5) we can show that the normalized temperature can be given by

$$t = \frac{\delta_{0c}^2 - 1}{\ln(\Gamma_c) - \ln(\delta_{0c})}. \tag{6}$$

So, for $t = 0.5$ and $\Gamma_c(t,\delta) = 2.0$, Eq. (5) will be valid only if $\delta_{0c} \approx 1.134$.

Now we proceed to the evaluation of the fracture toughness k_c. It can be defined by the work done to break the fiber bundle and is given by

$$K_c = \sum_i \tau_i, \tag{7}$$

where τ_i is the work done to break each fiber of the bundle. The work τ_i is obtained by the following expression:

$$\tau_i = \tfrac{1}{2} k z_i^2, \tag{8}$$

where z_i is the deformation of the fiber i.

Figure 3 shows the log-log plot of the fracture toughness K_c versus the force F for different system sizes. This figure shows how the fracture toughness k_c decreases with the increase of the force F, until it reaches a minimum value. This value is attained when a critical force F_c is applied to the system. Above of F_c the fiber bundle breaks catastrophically. It is known experimentally that a catastrophic break consumes little energy. As the force F decreases below F_c, the material can absorb more energy before it fractures and the number of cracks in the material increases. Also, notice that the fracture toughness increases with the system size. Thus, the higher the number of fibers the more energy will be absorbed by fracture.

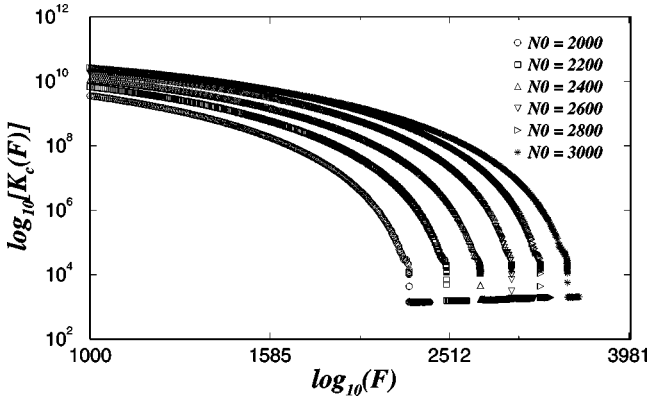

FIG. 3. Fracture toughness K_c vs the applied force F for different numbers of fibers N_0. The data were averaged over 1000 statistically independent samples.

Some works in the literature conjecture that there is a relationship between the fractal dimension and the fracture toughness [7–10], while others conjecture that such relationship does not exist [18,19]. In this paper we investigated the connection between the roughness W, directly related to the fractal dimension, and the toughness fracture K_c.

Figure 4 shows how the roughness W changes with the toughness K_c for several number of fibers N_0. In this figure we can see that the roughness W increases with the increase of the fracture toughness K_c, until it reaches a stationary state. We can interpret this figure in the following manner: with the increase of K_c, small cracks appear at all parts of the sample, making the fracture profile rougher. In the saturation region, we have a larger number of small cracks and they are totally uncorrelated. The phenomenon of saturation constitutes a finite size effect and is related to the number of segments in which the fibers are divided.

IV. CONCLUSION

In conclusion, we studied a model for fracture in fibrous materials in $(1+1)$ dimensions which take into account the

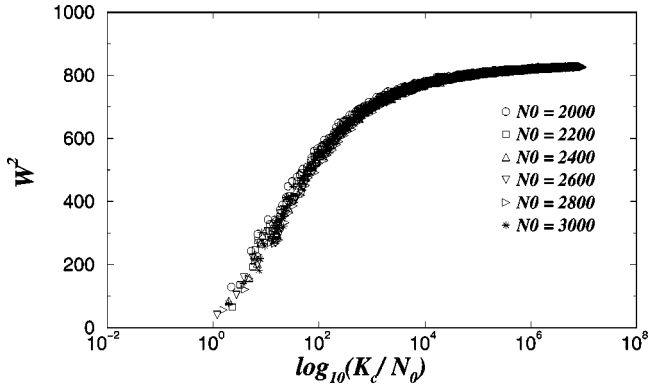

FIG. 4. Roughness W vs the fracture toughness K_c for different numbers of fibers N_0. The data were averaged over 1000 statistically independent samples.

rupture height of the fibers, in contrast with previous models. We obtained the fracture profile and evaluate its roughness and toughness. In this work we show that in the catastrophic regime the roughness of the fracture profile is reasonably smooth. In the shredding regime, in which slow cracks are formed in the material, the fracture profile is very rough. In this regime the energy necessary to break the material is higher than in the catastrophic regime. Our results indicate that the roughness W is related to the fracture toughness. We believe that the search for possible relationships between the roughness and the fracture toughness could stimulate further studies in order to check whether these relations are valid or not.

ACKNOWLEDGMENTS

We thank Marcelo Lobato Martins and Marcos da Silva Couto for helpful criticism of the manuscript. I.L.M. and A.T.B. acknowledge the kind hospitality of the Departamento de Física, UFMG. We also acknowledge the FAPEMIG (Brazilian agencies) for financial support.

[1] B.B. Mandelbrot, D.E. Passoja, and A.J. Paullay, Nature (London) **308**, 721 (1984).
[2] *Statistical Models for the Fracture of Disordered Media*, edited by H. J. Herrmann and S. Roux (North-Holland, Amsterdam, 1990).
[3] D. Sornette and C. Vanneste, Phys. Rev. Lett. **68**, 612 (1992).
[4] G.G. Batrouni and A. Hansen, Phys. Rev. Lett. **80**, 325 (1998).
[5] H.E. Daniels, Proc. R. Soc. London, Ser. A **183**, 404 (1945).
[6] I.L. Menezes-Sobrinho, J.G. Moreira, and A.T. Bernardes, Eur. Phys. J. B **13**, 313 (2000).
[7] J.A. Rodrigues, and V.C. Pandolfelli, Mater. Res. **1**, 47 (1998).
[8] S.Z. Zhang and C.W. Lung, J. Phys. D **22**, 790 (1989).
[9] G. Pezzotti, M. Sakai, Y. Okamoto, and T. Nishida, Mater. Sci. Eng., A **197**, 109 (1995).
[10] Z.Q. Mu and C.W. Lung, J. Phys. D **21**, 848 (1988).
[11] A.W. Thompson and M.F. Ashby, Scr. Metall. **18**, 127 (1984).
[12] I.L. Menezes-Sobrinho, J.G. Moreira, and A.T. Bernardes, Int. J. Mod. Phys. C **9**, 851 (1998).
[13] S.D. Zhang and E.J. Ding, Phys. Lett. A **193**, 425 (1994).
[14] R. da Silveira, Phys. Rev. Lett. **80**, 3157 (1998).
[15] A.T. Bernardes and J.G. Moreira, Phys. Rev. B **49**, 15 035 (1994).
[16] S.D. Zhang, Phys. Rev. E **59**, 1589 (1999).
[17] J.G. Moreira, J. Kamphorst Leal da Silva, and S. Oliffson Kamphorst, J. Phys. A **27**, 8079 (1994).
[18] E. Bouchaud, G. Lapasset and J. Planés, Europhys. Lett. **13**, 73 (1990).
[19] J. Schmittbuhl, S. Roux, and Y. Berthaud, Europhys. Lett. **28**, 585 (1994).